



UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	: DGE 2123
COURSE	: MATHEMATICS II
SEMESTER/SESSION	: 1 – 2023/2024
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **FIVE (5)** questions in SECTION A, **THREE (3)** questions in SECTION B and **TWO (2)** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)**INSTRUCTION: ANSWER ALL QUESTIONS.****QUESTION 1**Differentiate each of the following function with respect to x .

a) $y = 10x^4 - 3x^3 + 6x$ (2 marks)

b) $y = e^{3x} + \frac{2}{e^{2x}}$ (2 marks)

c) $y = 5 \sin(x) + \cos(3x)$ (2 marks)

d) $y = e^{x^2} + \ln(8x)$ (2 marks)

e) $y = \tan\left(\frac{x}{2}\right) + \sqrt{x}$ (2 marks)

QUESTION 2

Integrate each of the following function.

a) $\int (5x^4 - 2x + 3) dx$ (2 marks)

b) $\int \left(\sin(x) + \frac{1}{2} \sec^2(6x) \right) dx$ (2 marks)

c) $\int (4e^{2x-1} + e^x) dx$ (2 marks)

d) $\int \left(\frac{1}{x} - \frac{10}{5x+2} \right) dx$ (2 marks)

e) $\int \left(\frac{8}{x^4} + \frac{5}{e^{4x}} \right) dx$ (2 marks)

QUESTION 3

Find $\frac{dy}{dx}$ for the following function using the indicated techniques.

a) $y = x^{10} \ln x$ (use Product Rule) (3 marks)

b) $y = \frac{e^x + 2}{x^5}$ (use Quotient Rule) (3 marks)

c) $y = (3 + 2 \ln x)^8$ (use Chain Rule) (3 marks)

QUESTION 4

Integrate each of the following function using the given techniques.

a) $\int \frac{4x}{(8-x^2)^3} dx$ (use By Substitution method) (4 marks)

b) $\int 12x \cos(3x) dx$ (use By Parts method) (4 marks)

QUESTION 5

a) Find $\frac{dy}{dx}$ for $x^2 + 5 \sin y - 9 = 8xy$ by implicit differentiation. (5 marks)

b) Show that the following differential equation is exact.

$$(3x^2y + xy^2)dx + (x^3 + x^2y)dy = 0 \quad (3 \text{ marks})$$

c) Solve the following second order linear homogeneous differential equation (ODE).

i. $y'' - 25y = 0$ (2 marks)

ii. $y'' - 8y' + 33y = 0$ (3 marks)

SECTION B (30 MARKS)**INSTRUCTION: ANSWER ALL QUESTIONS.****QUESTION 1**

Solve $\int_2^3 \frac{6}{(x-1)(2x+1)} dx$. (7 marks)

QUESTION 2

a) Solve $\frac{dy}{dx} = 10e^{x-2y}$ using separation of the variables. (5 marks)

b) Find a general solution of the differential equation, $x \frac{dy}{dx} + 5y = x^4$ using the integrating factor method. (6 marks)

QUESTION 3

Consider the following second order linear non-homogeneous differential equation (ODE).

$$y'' - 5y' + 4y = 8x^2 - 1$$

a) Find the complementary function, y_c . (3 marks)

b) Find the particular function, y_p . (8 marks)

c) Find the general solution of the given differential equation above. (1 mark)

SECTION C (20 MARKS)**INSTRUCTION: ANSWER ALL QUESTIONS.****QUESTION 1**

- a) The radius of a spherical balloon is increasing at a rate of 3 cm/min. Find the rate of volume of the balloon when the radius is 5 cm. (The formula for the volume of the spherical balloon is $V = \frac{4}{3}\pi r^3$) (3 marks)
- b) Find the stationary point(s) on the curve equation, $y = x^3 + \frac{1}{2}x - 10x + 5$ and determine their nature. Then, sketch the graph. (10 marks)

QUESTION 2

Find the area enclosed between the curve, $y = 4 - x^2$ and the line, $y = x - 2$. Sketch the graphs and label the shaded area. (7 marks)

----- END OF QUESTION -----

FORMULA

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(kf(x)) = k \frac{d}{dx} f(x)$$

$$\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sin f(x)) = f'(x) \cos f(x)$$

$$\frac{d}{dx}(\cos f(x)) = -f'(x) \sin f(x)$$

$$\frac{d}{dx}(\tan f(x)) = f'(x) \sec^2 f(x)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x) e^{f(x)}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} f'(x)$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int dx = x + C$$

$$\int k f(x) dx = k \int f(x) dx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C, \quad n \neq -1$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + C$$

$$\int \sin(ax+b) dx = -\frac{\cos(ax+b)}{a} + C$$

$$\int \sec^2(ax+b) dx = \frac{\tan(ax+b)}{a} + C$$

$$\int e^x dx = e^x + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int u dv = uv - \int v du$$

$$\text{Area} = \int_a^b [f(x) - g(x)] dx$$

$$\rho y = \int \rho Q(x) dx \text{ where } \rho = e^{\int p(x) dx}$$

TABLE OF PARTICULAR FUNCTION, y_p

Types of $g(x)$	Example of $g(x)$	General solution of y_p
Polynomial	$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$	$A_n x^n + A_{n-1} x^{n-1} + \dots + A_0$
Exponential	$e^{\alpha x}$	$A e^{\alpha x}$
Trigonometric	$a \sin \beta x$ $a \cos \beta x$ $a \sin \beta x + a \cos \beta x$	$A \cos \beta x + B \sin \beta x$
Combination of types i,ii,iii	$e^{\alpha x} + x^2$ $e^{\alpha x} + a \sin \beta x$	$A e^{\alpha x} + B x^2 + C x + D$ $A e^{\alpha x} + B \cos \beta x + C \sin \beta x$

